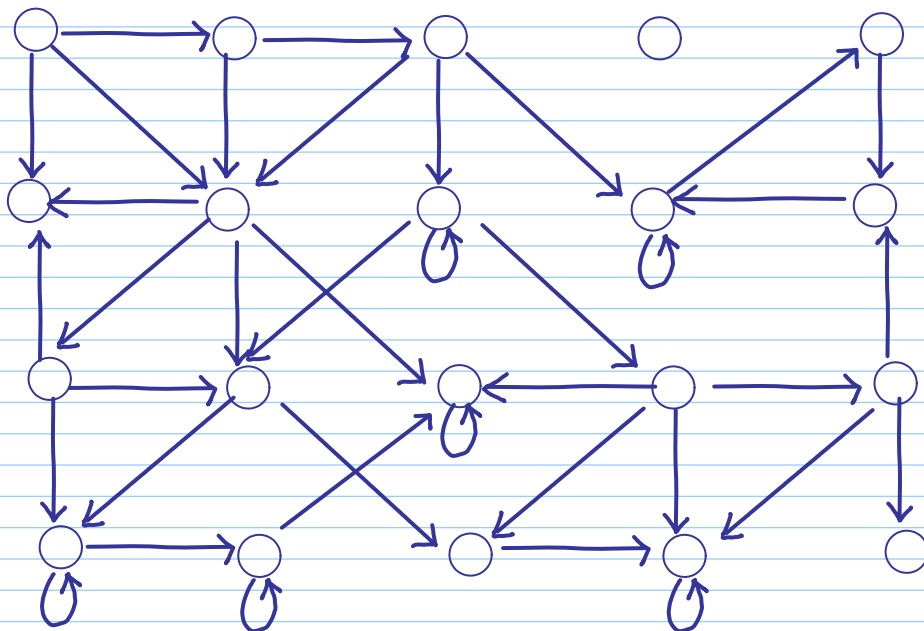


Choosing Data Structures

Note Title

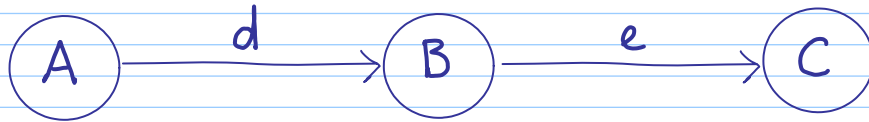
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Example: Graph Searching



A **path** from node s to node t is an alternating sequence of nodes and edges that begins with s and ends with t .

Examples:



Notation:

$u \rightarrow v$

edge from u to v

$u \rightarrow^* v$

path from u to v

Conventions:

s, t, u, v :: node

U, V :: set of node

e :: edge

p :: path

Definitions

$\text{reachable}.u = \{p, v : p \text{ is from } u \text{ to } v : v\}$

$\text{reachable}.U = \{u, p, v : u \in U \wedge p \text{ is from } u \text{ to } v : v\}$

$\text{directly_reachable}.u$

$= \{e, v : e \text{ is from } u \text{ to } v : v\}$

$\text{directly_reachable}.U$

$= \{u, e, v : u \in U \wedge e \text{ is from } u \text{ to } v : v\}$

{true}

reached :: set of node

reached := {s}

{Invariant : reachable.{s} = reachable.reached

Bound fn. : |reached| }

while reached changes do

reached := reached \cup directly_reachable.reached

{ reached = reachable.{s} }

Split reached into black \cup grey

black, grey, white := \emptyset , {s}, N-{s} ;

{ Invariant : reachable.{s} = black \cup reachable.grey
 \wedge {black, grey, white} partitions N }

while grey $\neq \emptyset$ do

begin black := black \cup grey ;

grey := white \cap directly_reachable.grey ;

white := white $\cap \neg$ grey

end

{ reachable.{s} = black }

Replace operations on sets by operations on elements

black, grey, white := $\emptyset, \{s\}, N - \{s\}$;

{ Invariant : reachable. $\{s\}$ = black \cup reachable.grey
^ {black, grey, white} partitions N }

while grey $\neq \emptyset$ do

begin choose $u \in$ grey ;

remove u from grey and add it to black ;

for each v directly reachable from u do

if $v \in$ white then

add v to grey and remove it from white

end

{ reachable. $\{s\}$ = black }

Choose data structures :

Set	Operation	Frequency
black	$:= \emptyset$ add	1 $\leq N $
grey	$:= \{s\}$ choose and remove add	$\leq n$
white	$:= N - \{s\}$ test for membership remove	1 $\leq E $ $\leq N $